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# A Bayesian Analysis of a Multiplicative Treatment Effect in Weather Modification

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Bayesian techniques are developed to investigate a multiplicative treatment effect when data samples on a control and a treatment are available. The random variables are assumed to follow highly skewed gamma distributions with the same shape parameters. The data sample is from a randomized airborne pyrotechnic seeding experiment on 52 isolated cumulus clouds in south Florida, half of which were massively injected with silver iodide smoke and the other untreated half were identically observed. The physical hypothesis is that massive seeding can, under specified conditions, invigorate cloud growth and thereby cause increased precipitation. The random variables compared are seeded and unseeded rain volume, measured by means of calibrated radar.

## KEY WORDS

Bayesian Analysis  
Gamma Distribution  
Multiplicative Effect  
Weather Modification

## 1. INTRODUCTION

The statistical analysis of weather modification experiments requires, in many instances, a multiplicative treatment effect model in the presence of a highly skewed distribution. The present article develops a Bayesian analysis based on the above conditions when two independent data sets are available, one giving independent replications on a control and the other independent replications on a treatment. The purpose of the experimentation and analysis is to obtain information concerning the most probable values of the multiplicative effect and, if possible, to make a definitive statement about whether there is a positive treatment effect.

The analysis and experimental data are the result of a series of weather modification experiments con-

ducted in south Florida from 1968 to 1972. These experiments were designed to test a hypothesis called "dynamic seeding" in which it is postulated that massive silver iodide seeding can, under specified conditions, lead to invigorated cumulus growth and prolonged lifetimes, thereby increased processing of water, and augmented precipitation. The experiments were guided by a numerical simulation of cumulus growth, [7, 8] which specified suitable conditions in terms of a "seedability" parameter, defined as the difference in predicted maximum height of seeded versus unseeded clouds. The design execution and meteorological evaluation of the experiments have been published elsewhere [9, 10, 11]. The experimental unit in this series was an isolated, growing cumulus cloud. The random variable studied here is the rain volume falling from the cloud after the "seeding" run, measured by means of calibrated radar. "Seeding" runs were made in both seeded and control clouds since the experimenters were unaware whether the flare rack was armed and therefore whether or not the cloud was undergoing seeding.

In numerous studies concerned with the measurement of precipitation, it has been found that the

data are well described by the gamma distributions [1, particularly bibliography].

With the current sample (see Table 1), it was shown that a gamma distribution with a shape parameter less than one was a satisfactory fit to the data [6]. There is an indication [2] that the distribution of rainfall amounts may be slightly more "heavy tailed" than the gamma distribution allows. However, the relatively small data sets restrict any definite conclusion. The hypothesized multiplicative treatment effect is supported by the quantile-quantile plot of Figure 1. The value 1 has been added to the pairs of ordered data for display purposes. Since the shift of the points from the line  $Y = X$  could be described by a parallel line, the plot lends support to the multiplicative assumptions. Moreover, the coefficients of variation for the control and seeded data sets, 1.34 and 1.25 respectively, are approximately equal supporting the assumption that the treatment effect is multiplicative.

Previous investigators have used two-sample non-parametric techniques [5, 11], data transformations to obtain approximate normality and/or an additive effect and an asymptotically optimal  $C(\alpha)$  test developed by Neyman and Scott [3] under the assumptions utilized here.

2. EXPERIMENTAL CONDITIONS AND BASIC MODEL

In any experiment it is necessary to define the basic unit upon which a treatment may be applied and the appropriate measurement is to be taken. As applied here, the experimental unit is defined in terms of preset conditions intended to make the units as homogeneous as possible. In south Florida, days with satisfactory model-predicted "seedability" (potential seeded growth greater than about 1km)

are generally fair weather conditions with adequate numbers of apparently similar isolated cumulus clouds growing and dissipating. A cloud is selected as a "go" cloud if its top is rising, is at a height between about 5.5 and 8 km, and has a liquid water content greater than about  $0.5 \text{ gm } m^{-3}$  in the cloud droplet size range. The physical rationale of these criteria have been extensively described in the meteorological literature [9, 10, 11].

Once an experimental unit is determined to be eligible for the experiment, a randomized decision is made to determine if the unit is to be subjected to the treatment or left as a control. In either case, the appropriate set of observations are taken. Let the random variable  $X$  represent the response measured on an experimental unit designated to receive the control, and let the random variable  $Y$  represent the response measured on an experimental unit designated to receive the treatment. In this analysis, the response in question is the total rain volume falling from cloud base following the "seeding" penetration by the aircraft. Numerous other variables were measured to test parts of the physical hypothesis and the numerical simulation; these are described elsewhere [10, 11]. The resulting set of data to be analyzed here consists of 26 control observations and 26 seeded, i.e. treated observations.

Let the control observations be designated as  $\mathbf{x} = (x_1, \dots, x_m)$  assumed to be drawn from a population with the gamma density

$$p(x/\alpha, \beta_c) = \frac{\beta_c^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta_c x}$$

where  $\alpha > 0, \beta_c > 0, x > 0$ . Similarly let the seeded observations be  $\mathbf{y} = (y_1, \dots, y_n)$  from a gamma

TABLE 1—Single Cloud Data for 1968 and 1970

Rain Volume for Total Cloud Lifetime (acre-ft)					
Seeded Clouds			Control Clouds		
129.6	17.5	1697.8	26.1	321.2	4.9
31.4	200.7	334.1	26.3	68.5	4.9
2745.6	274.7	118.3	87.0	81.2	41.1
489.1	274.7	255.0	95.0	47.3	29.0
430.0	7.7	115.3	372.4	28.6	163.0
302.8	1656.0	242.5	0.0(1)	830.1	244.3
119.0	978.0	32.7	17.3	345.5	147.8
4.1	198.6	40.6	24.4	1202.6	21.7
92.4	703.4		11.5	36.6	

population with the same shape parameter  $\alpha$  but with a possibly different scale parameter  $\beta_s$ .

If the expected values of  $X$  and  $Y$  are given by  $\mu_c = \alpha/\beta_c$  and  $\mu_T = \alpha/\beta_T$ , then the parameter of interest to the experimenter is defined as  $\theta = \mu_T/\mu_c$ . In terms of the scale parameters of the distributions  $\beta_T = \beta_c/\theta$  so that  $\theta$  corresponds to a scale change. Moreover the distribution of  $\theta X$  is the same as the distribution for  $Y$ . Under these conditions  $\theta$  is the factor by which hypothetically, the treatment multiplies the average response that would occur if the treatment had not been applied to the experimental unit. Thus,  $\theta = 1$  means no treatment effect, while  $\theta = 1.4$  means a 40 per cent increase due to the treatment.

From the definition of  $\theta$  and its connection to the scale parameters, the probability model can be parameterized in the more convenient form

$$p(y/\alpha, \mu_c, \theta) = \frac{\alpha^\alpha y^{\alpha-1} e^{-(\alpha y/\theta \mu_c)}}{\Gamma(\alpha) \mu_c^\alpha \theta^\alpha}, \quad y > 0$$

The control density is the special case when  $\theta = 1$ .

This formulation for the parameters is useful since it directly incorporates the parameter  $\theta$  of interest to the meteorologist and the parameter  $\mu_c$  is in a form that allows the meteorologist to assess his prior knowledge concerning the parameter.

The likelihood function is easily seen to be proportional to

$$l(\mathbf{x}, \mathbf{y}/\alpha, \mu_c, \theta)$$

$$\propto \frac{\alpha^{N\alpha} \exp \left\{ [mg_x + ng_y]\alpha - \bar{x} \frac{m\alpha}{\mu_c} - \bar{y} \frac{n\alpha}{\theta \mu_c} \right\}}{\Gamma(\alpha)^N \mu_c^{N\alpha} \theta^{n\alpha} \exp \{mg_x + ng_y\}}$$

where  $\bar{x}, \bar{y}$  are the arithmetic sample means;  $g_x, g_y$  are the arithmetic sample means of the natural logarithms of the data; and  $N = n + m$ . Furthermore, the set  $\{mg_x + ng_y, \bar{x}, \bar{y}\}$  forms a sufficient statistic for the parameters  $(\alpha, \mu_c, \theta)$ . If  $\alpha$  is known, then the sample means are sufficient for  $(\mu_c, \theta)$  so that the assumption of  $\alpha$  being known effectively says that the dispersion of the distributions is known.

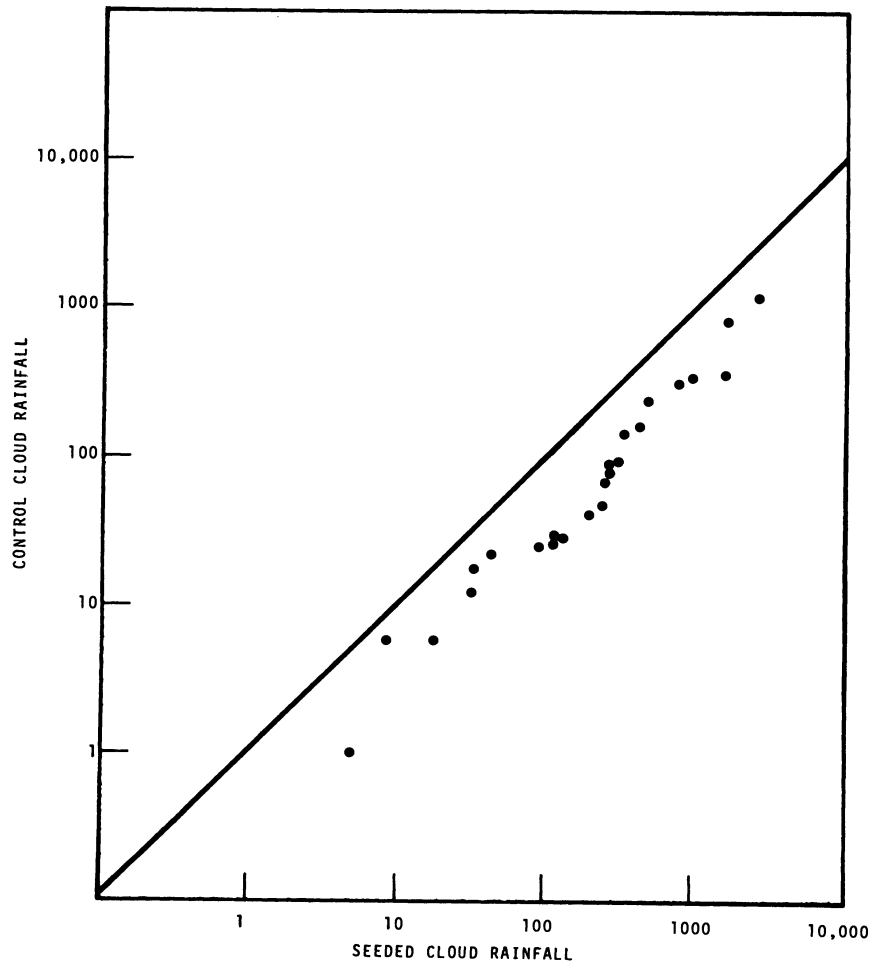


FIGURE 1—Quantile-Quantile logarithmic plot of seeded cloud rainfall versus control cloud rainfall.

3. ASSESSMENT OF PRIOR DENSITY FOR  $(\alpha, \mu_c, \theta)$  AND POSTERIOR ANALYSIS

In assessing the joint prior density for  $(\alpha, \mu_c, \theta)$  the major assumption is that the parameters are independent under the experimental conditions previously described. For  $\mu_c$  and  $\theta$ , while it is true that the same seeding effect is not expected for "wet" clouds and "dry" clouds, the "seedability" criterion was designed to select a middle class of clouds for which one would expect the dynamic seeding to have an effect and that this effect would be a constant multiplicative effect.

The prior selected is the joint improper prior  $p(\alpha, \mu_c, \theta) \propto 1/\theta 1/\mu_c \quad \theta > 0, \mu_c > 0, 0.1 \leq \alpha \leq 3.0$ .

This selection represents diffuse or "vague" prior information for  $\mu_c$  and  $\theta$  and a locally flat prior for  $\alpha$  in the range  $[0.1, 3.0]$ . The priors selected for  $\theta$  and  $\mu_c$  result in uniform priors for  $\log \theta$  and  $\log \mu_c$  and are in some sense diffuse.

The posterior density with this prior is proportional to

$$p(\alpha, \mu_c, \theta | \mathbf{x}, \mathbf{y})$$

$$\propto \frac{\alpha^{N\alpha} \exp \{[mg_x + ng_y]\alpha - m\alpha\bar{x}/\mu_c - n\alpha\bar{y}/(\theta\mu_c)\}}{\Gamma(\alpha)^N \mu_c^{N\alpha+1} \theta^{n\alpha+1}}$$

where  $\theta > 0, \mu_c > 0, 0.1 \leq \alpha \leq 3.0$ . Straightforward integrations can be performed, using knowledge of the gamma and beta integrals, to derive the

forms of the marginal densities of  $(\alpha, \theta)$  and  $\alpha$ . The marginal density of  $\theta$ , which is the one of interest, must be obtained numerically. This is readily accomplished using the marginal density

$$p(\alpha, \theta | \mathbf{x}, \mathbf{y}) \propto \frac{\exp \{[mg_x + ng_y]\alpha + \ln \Gamma(N\alpha) - N \ln \Gamma(\alpha)\}}{\theta^{n\alpha+1} (m\bar{x} + n\bar{y}/\theta)^{n\alpha}}$$

in connection with one and two dimensional numerical integration. The only difficulty in the integration is the necessity for an approximation to  $\ln \Gamma(a)$ , which may be found in most good mathematical handbooks.

Figure 2 shows the marginal posterior density for the seeding effect parameter  $\theta$ . The posterior mode is 2.4, the mean is 2.87 and the standard deviation is 1.10. The posterior indicates that there remains a great deal of uncertainty concerning a point estimate of the magnitude of the seeding effect. However, there is less than one percent of the posterior density below one and the 95% equal tail integrated probability interval is  $1.24 \leq \theta \leq 5.51$ . Based on this the authors conclude that there is a positive seeding effect for single clouds and a tentative estimate of its magnitude is the modal value 2.4.

Another marginal posterior density of interest is for the shape parameter  $\alpha$  shown in Figure 3. Note the symmetry and small variability around the modal value 0.6. In previous work it was assumed

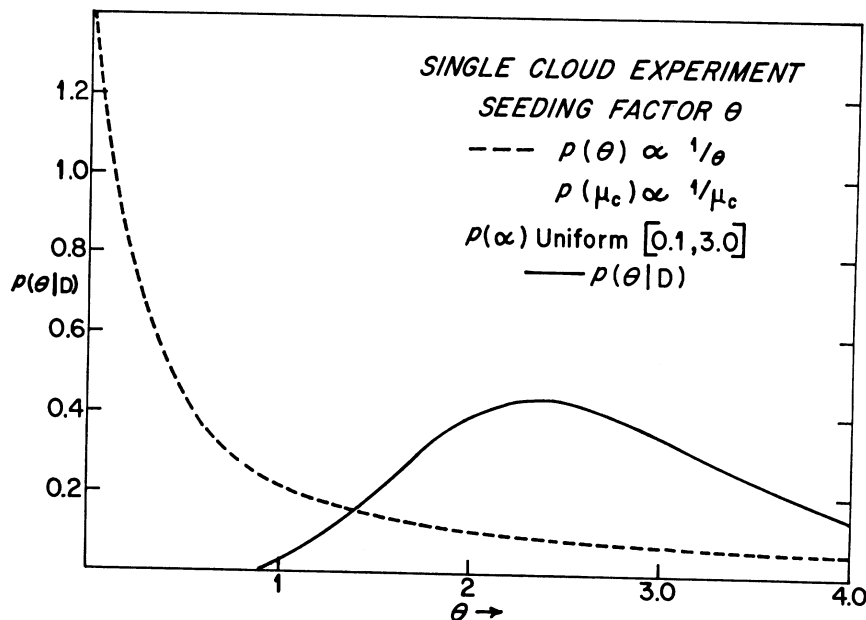


FIGURE 2—Posterior probability density of seeding factor  $\theta$ , given the experimental data. Prior probabilities are placed on three variables, namely  $\theta$  (proportional to  $1/\theta$ ), the control population mean  $\mu_c$  (proportional to  $1/\mu_c$ ) and the gamma shape parameter  $\alpha$  (uniform in the range 0.1 to 3.0, see Figure 3). The mean of the distribution is 2.87 standard deviation 1.10. The equal-tailed 95% integrated probability extends from  $\theta = 1.24$  to  $\theta = 5.51$ .

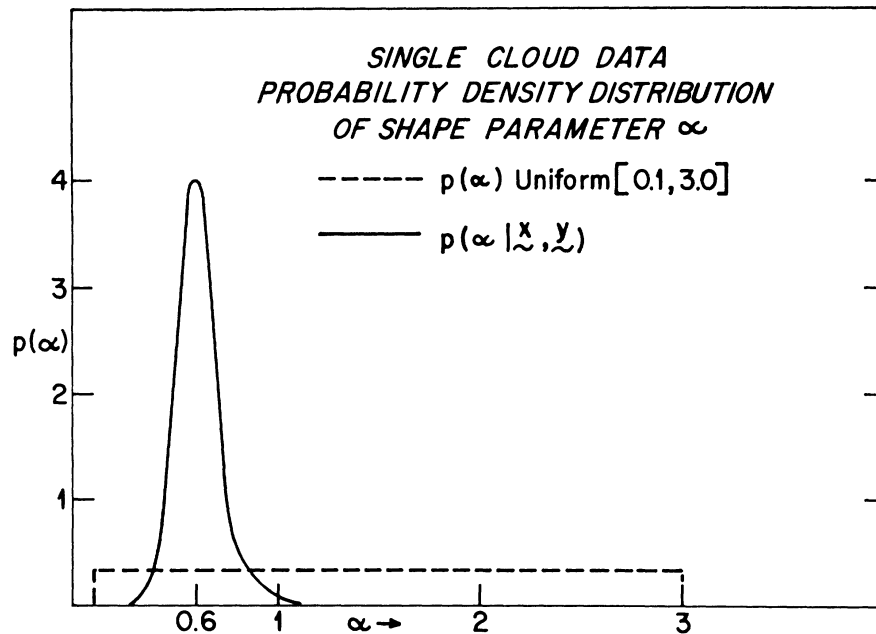


FIGURE 3—Posterior probability density of the gamma shape parameter  $\alpha$ , given the data of both samples. The assumed prior probability is uniform in the range 0.1 to 3.0.

that  $\alpha = 0.6$  was a known parameter. This assumption, which makes the analysis much simpler, resulted in only a minor decrease in variability compared with the present analysis. A more detailed discussion of this previous work is contained in Olsen [4].

#### 4. SUMMARY AND CONCLUSIONS

A Bayesian approach has been given for the analysis of a multiplicative treatment effect when two samples are available from a highly skewed gamma distribution. A more meaningful parameterization was developed and the analysis of a set of weather modification data was given. It was concluded that there was a positive seeding effect with a magnitude of 2.4.

In terms of amount of water, the seed-control rainfall differences measured in this experiment were quite large, the difference averaging  $3.3 \times 10^5$  m<sup>3</sup> per cloud. The multiplicative seeding factor of about 2-3, is the largest that has been demonstrated in a rain modification experiment.

Nevertheless, the experiment was primarily scientific in objective and was intended to have long-range rather than direct application to water resources augmentation in Florida. The reason is that isolated cumulus clouds, the subjects of this experiment, do not provide a large fraction of Florida precipitation, which is produced in much greater magnitude when two or more of these cumuli join to become a "merger complex." "Mergers" com-

monly produce 5-10 times the water volume as does an isolated large cumulus.

Experiments using dynamic seeding in the much more complex problem of attempting to induce merger and determining whether multiple cloud treatment can augment rainfall over a sizable area began in Florida in 1970 [9]. The sample size is not yet adequate in 1974 for a definitive estimate of seeding factors.

#### 5. ACKNOWLEDGMENTS

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